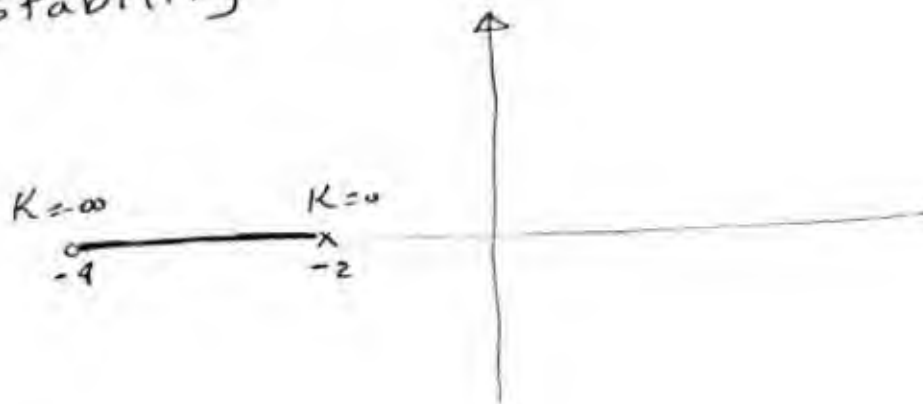


# Control Lec 1

Quiz  $GH(s) = \frac{K(s+4)}{(s+2)^3}$

→ Draw Root locus • find range of  $K$  for stability

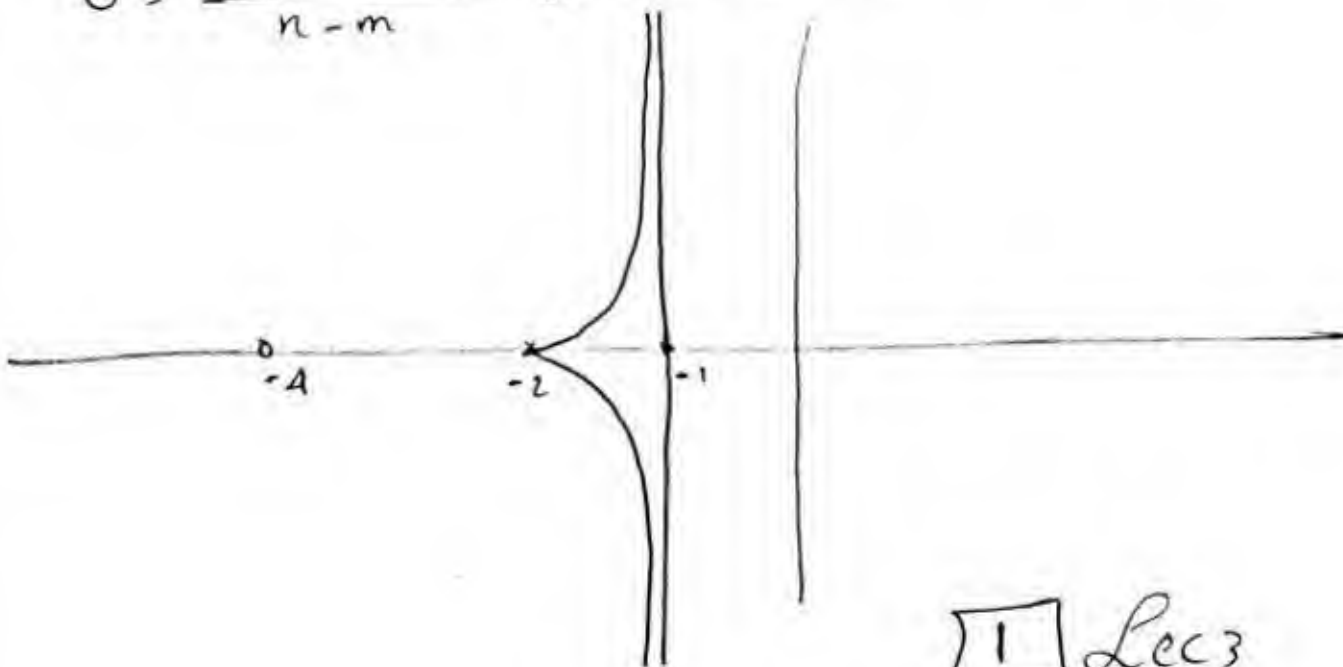


## Asymptotes

$$\text{number} = 3 - 1 = 2$$

$$C_A = \frac{(-2-2-2) - (-4)}{-2} = -1$$

$$\theta = \frac{(2L+1)180^\circ}{n-m} \Rightarrow \theta_1 = 90^\circ, \theta_2 = -90^\circ$$



[EX]

$$GH(s) = \frac{K}{s(s^2 + 4s + 13)}$$

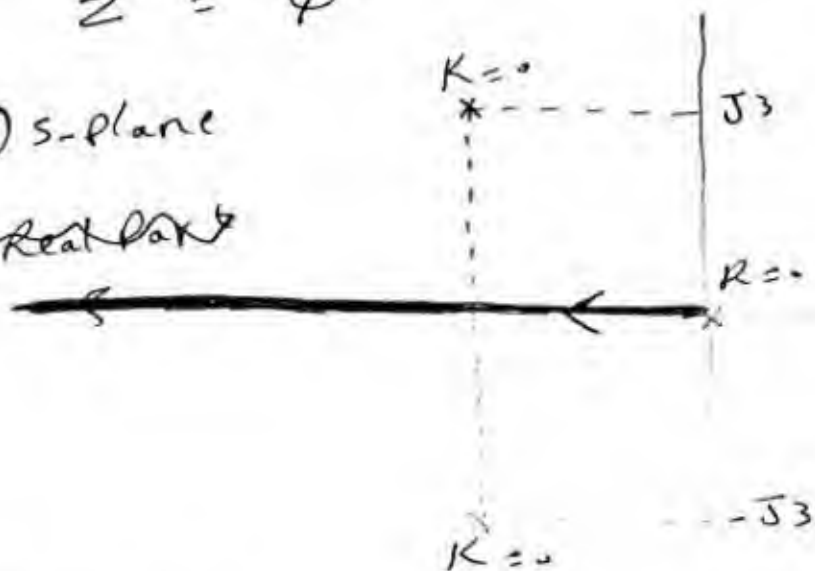
$$s_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \times 13}}{2}$$

① Poles  $\Rightarrow 0, -2 + j3, -2 - j3$

$$Z = \emptyset$$

② s-plane

③ Real Part



③ Real Part  $\Rightarrow 0 \rightarrow -\infty$

[4] Asymptotes

$$\rightarrow \text{number} = 3 - 0 = 3$$

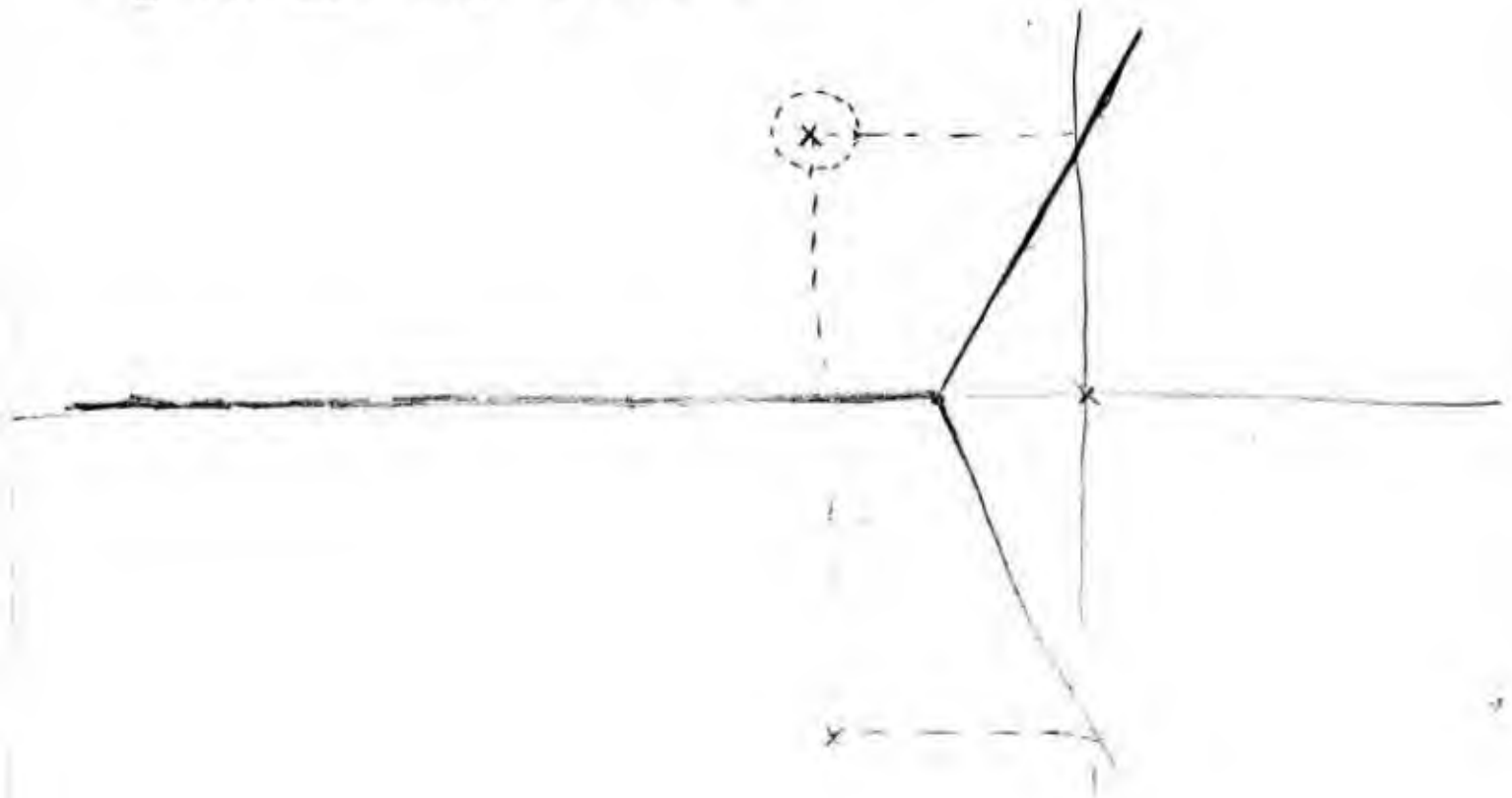
$$\rightarrow C_A = \frac{(0 + j3 - 2 - 2 - j3) - (0)}{3} = \frac{-4}{3} = -1.33$$

$$\Theta = \frac{(2L+1)180}{3}$$

$$L=0 \Rightarrow \Theta_1 = 60$$

$$L=1 \Rightarrow \Theta_2 = 180$$

$$L=2 \Rightarrow \Theta_3 = 300 = -60$$

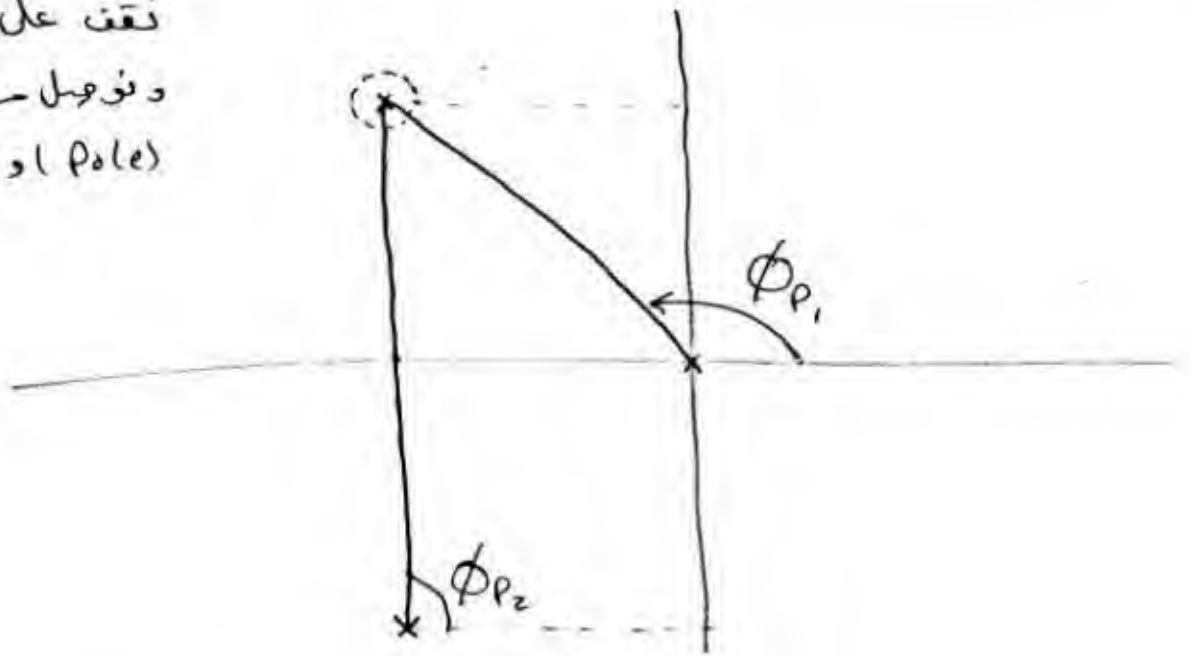


⑤ Departure Angle

$$\Theta_D = 180 - \phi_p + \phi_z$$

مجموع زوایا

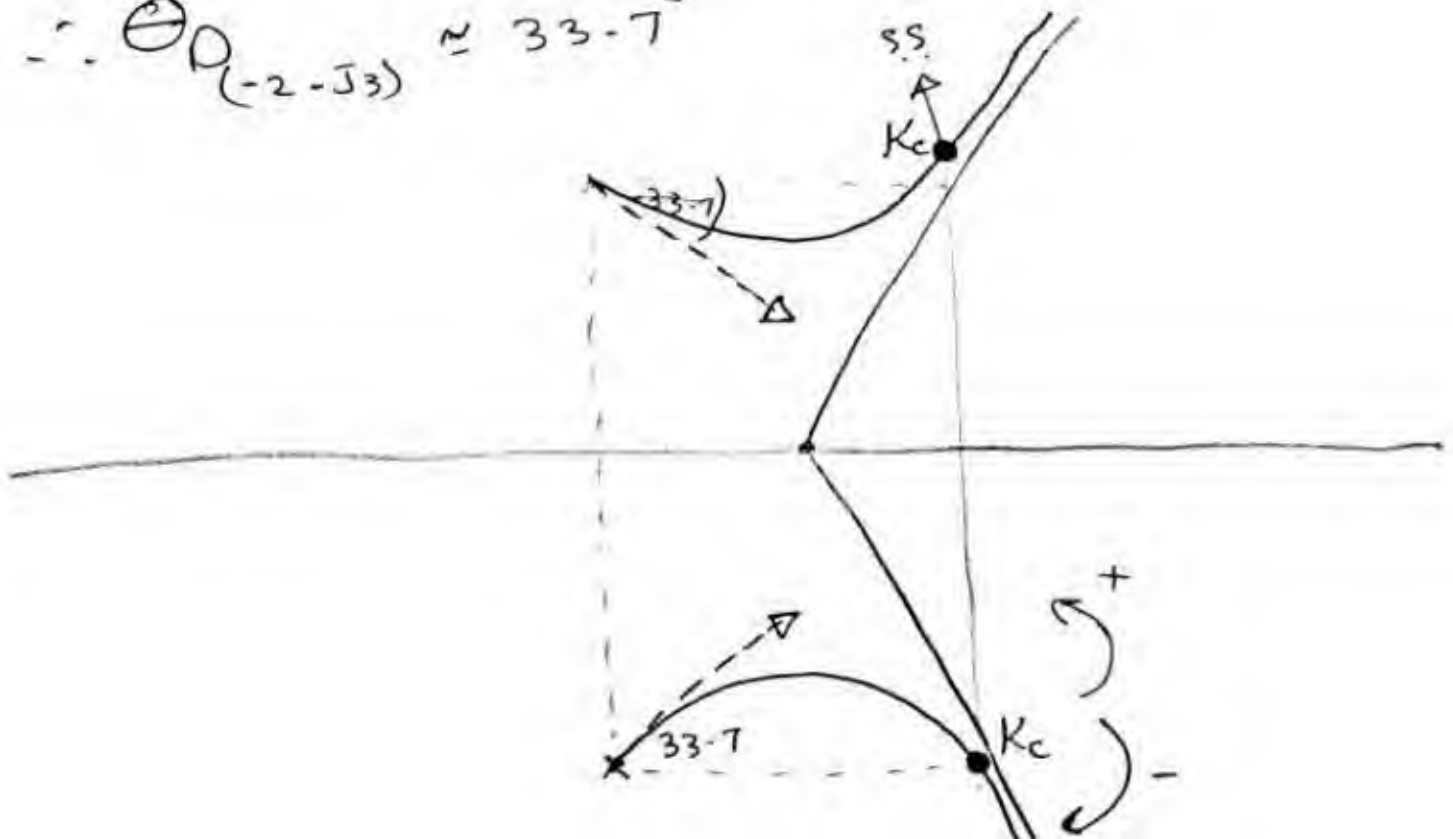
نقطه علی کل (Pole)  
و نقطه صفر خط لکل  
(Zero) و (Pole)



$$\theta_{D_{(-2+J3)}} = 180 - (\phi_{P_1} + \phi_{P_2}) + 0$$

$$= 180 - \left[ \left( 180 - \tan^{-1}\left(\frac{3}{2}\right) \right) + 90 \right] \approx -33.7^\circ$$

$$\therefore \theta_{D_{(-2-J3)}} \approx 33.7^\circ$$



[6] At imag. Axis

ch. eq  $\Rightarrow 1 + G H(s) = 0$

$$1 + \frac{K}{s^3 + 4s^2 + 13s} = 0$$

$$s^3 + 4s^2 + 13s + K = 0$$

$$\frac{52 - K}{4} \geq 0$$

$$52 - K \geq 0$$

$$K \leq 52 \rightarrow (1)$$

$$K \geq 0 \rightarrow (2)$$

$s^3$	1	13	
$s^2$	4	K	
$s^1$	$\frac{52 - K}{4}$	$\geq 0 \rightarrow (1)$	
$s^0$	K	$\geq 0 \rightarrow (2)$	

range of stability for  $K$   $0 < K < 52$

التي يتغير فيه جذور النظام على المحور الحقيقي

$$K_{critical} = 52$$

$$A(s) = 4s^2 + K_c = 0$$

$$4s^2 + 52 = 0$$

$$s = \pm j3.6$$

[5] Lec 3

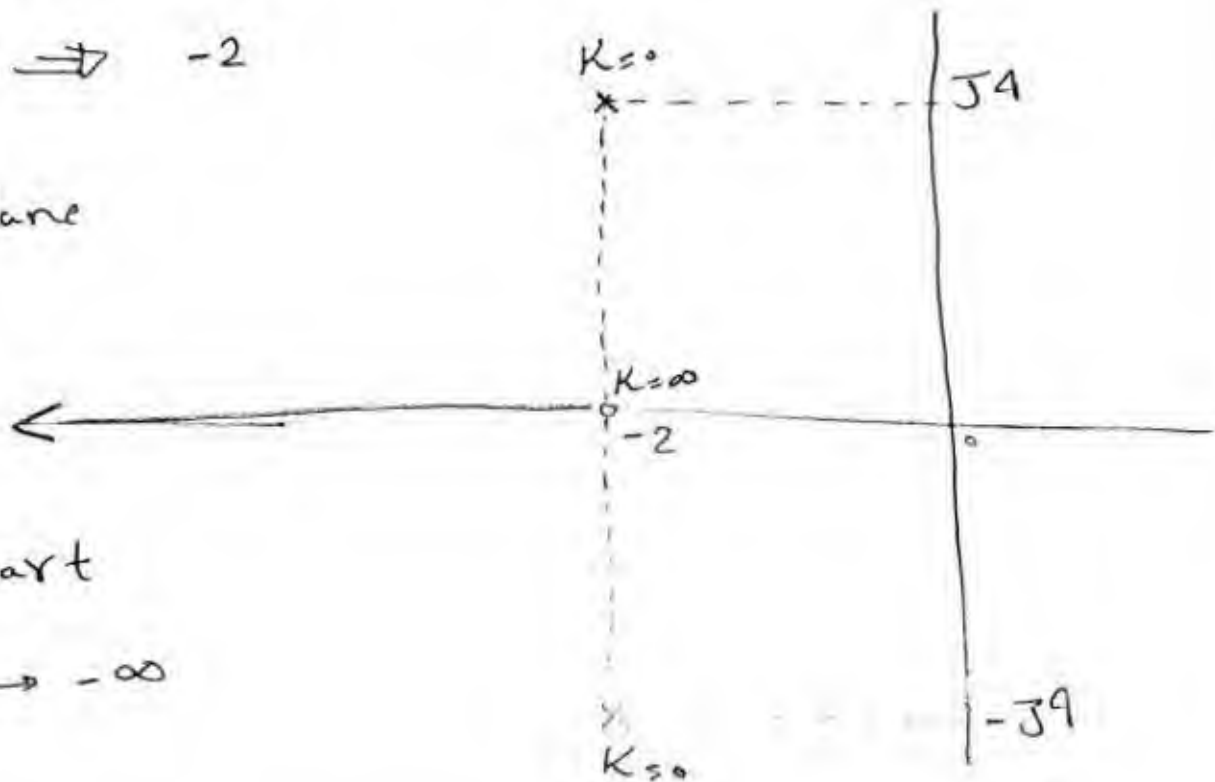
Ex

$$G H(s) = \frac{K(s+2)}{s^2 + 4s + 20}$$

① Poles  $\Rightarrow -2 + j4$  &  $-2 - j4$

Zeros  $\Rightarrow -2$

② s-plane



③ real part

$-2 \rightarrow -\infty$

4] Breaking Point

ch. eqn

$$1 + G H(s) = 0$$

$$G H(s) = -1$$

$$\frac{K(s+2)}{s^2 + 4s + 20} = -1$$

$$K = - \left[ \frac{s^2 + 4s + 20}{s + 2} \right] \rightarrow *$$

$$\frac{dK}{ds} = 0 \quad s = \frac{(s+2)(2s+4) - (s^2 + 4s + 20)(1)}{(s+2)^2}$$

$$(s+2)(2s+4) - (s^2 + 4s + 20) = 0$$

$$2s^2 + 8s + 8 - s^2 - 4s - 20 = 0$$

$$s^2 + 4s - 12 = 0$$

$$(s - 2)(s + 6) = 0$$

$$s = 2 \quad \times \times \quad s = -6 \quad \checkmark \checkmark$$

in \*

Breaking point ( $s_b$ ) = -6

$$K \Big|_{s_b = -6} \quad s \quad 8$$

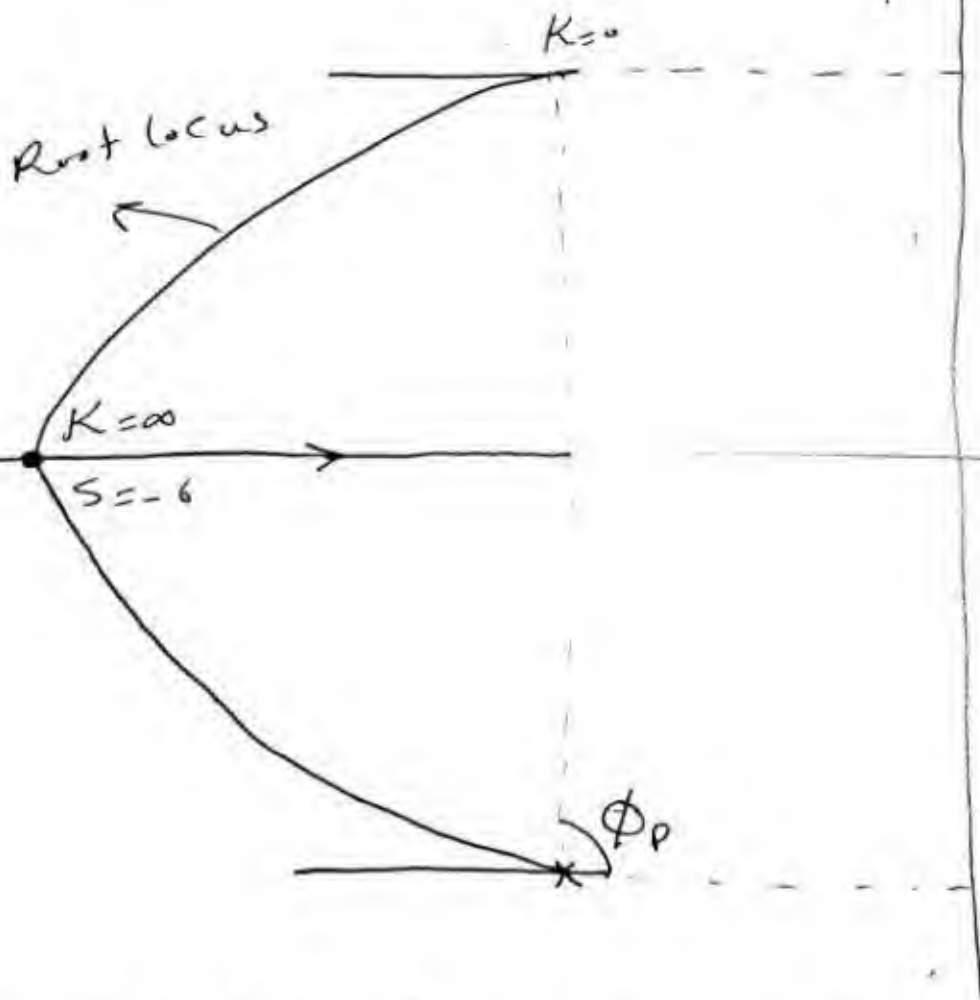
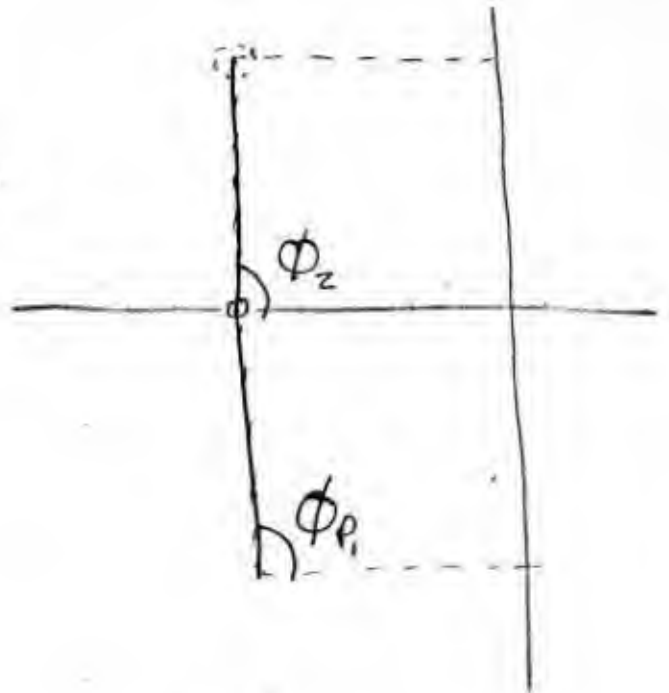
□ Page 3

# 5 Dep. Angle

$$\phi_{0_{-2+J4}} = 180 - 90 + 90$$

$$\phi_{0_{-2+J4}} = 180^\circ$$

$$\phi_{0_{-2-J4}} = -180^\circ$$



→ stable for all  $K \geq 0$



$$\boxed{\text{Ex}} \quad GH(s) = \frac{20(1+Ks)}{s(s+1)(s+4)}$$

ch. eqn  $1 + GH(s) = 0$   $s^2 + 5s - 4$

$$1 + \frac{20(1+Ks)}{s(s+1)(s+4)} = 0$$

$$s(s+1)(s+4) + 20 + 20Ks = 0$$

$$1 + \frac{20Ks}{s(s+1)(s+4) + 20} = 0$$

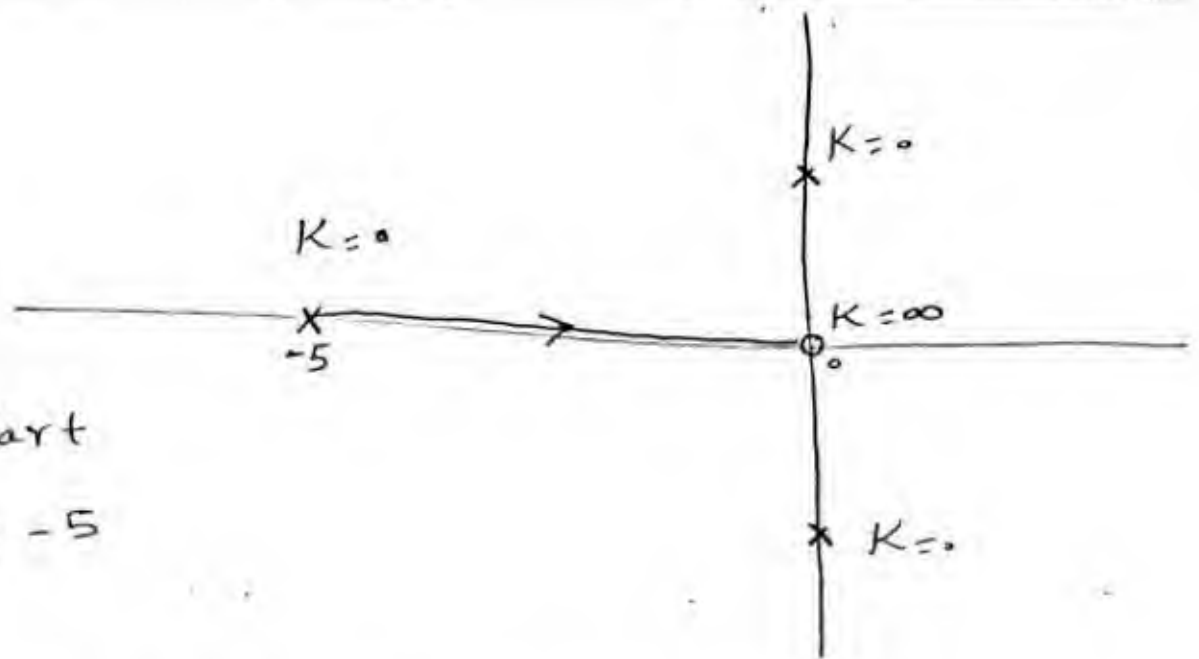
$$1 + GH(s) = 0$$

$$GH(s) = \frac{20Ks}{s(s+1)(s+4) + 20}$$

① Poles  $\Rightarrow -5, j2, -j2$

Zeros  $\Rightarrow 0$

$\boxed{a} \quad P$



③ Real Part

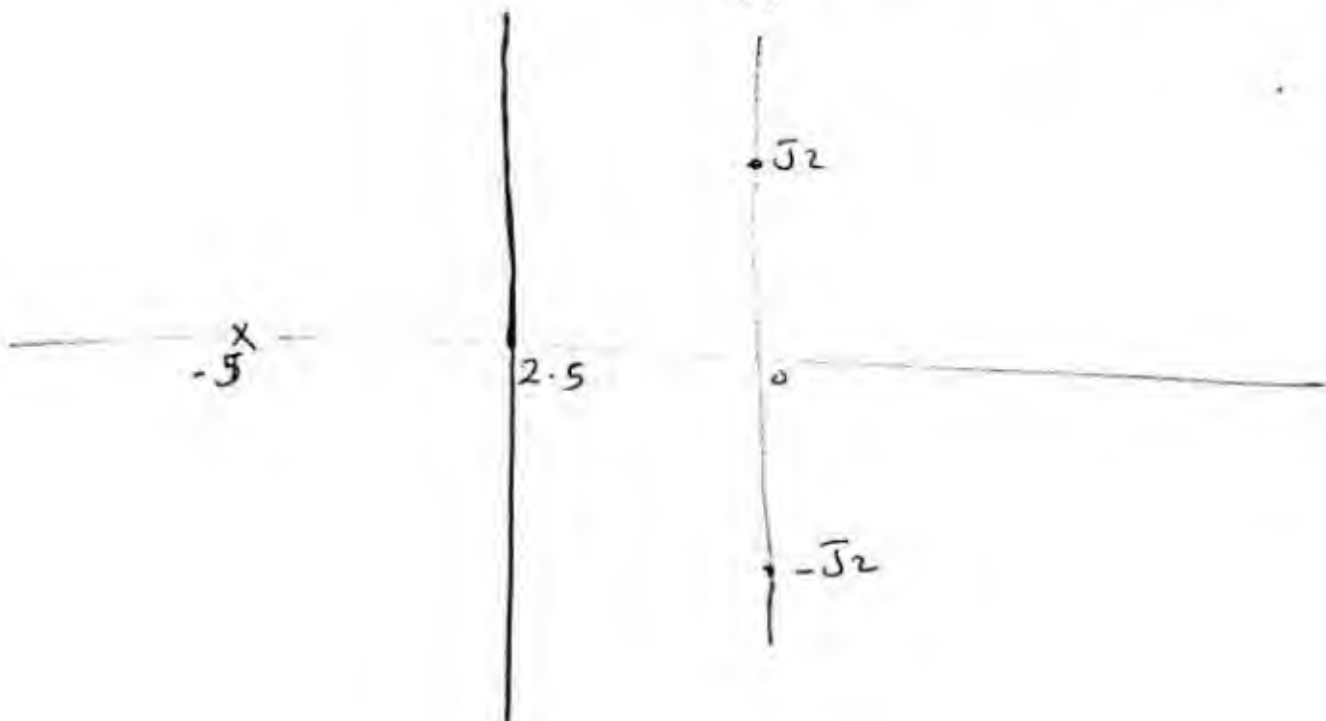
$$0 \rightarrow -5$$

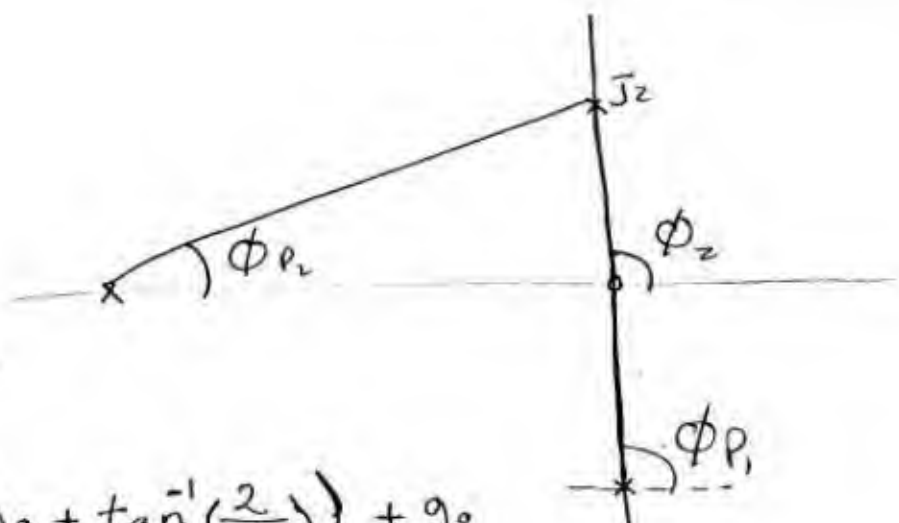
④ Asymptotes

$$\rightarrow \text{① number} = 3 - 1 = 2$$

$$\rightarrow \text{② } C_A = \frac{(-5 + j2 - j2) - 0}{2} = \frac{-5}{2} = -2.5$$

$$\rightarrow \theta = \frac{(2L+1)180}{2} = 90^\circ, -90^\circ$$



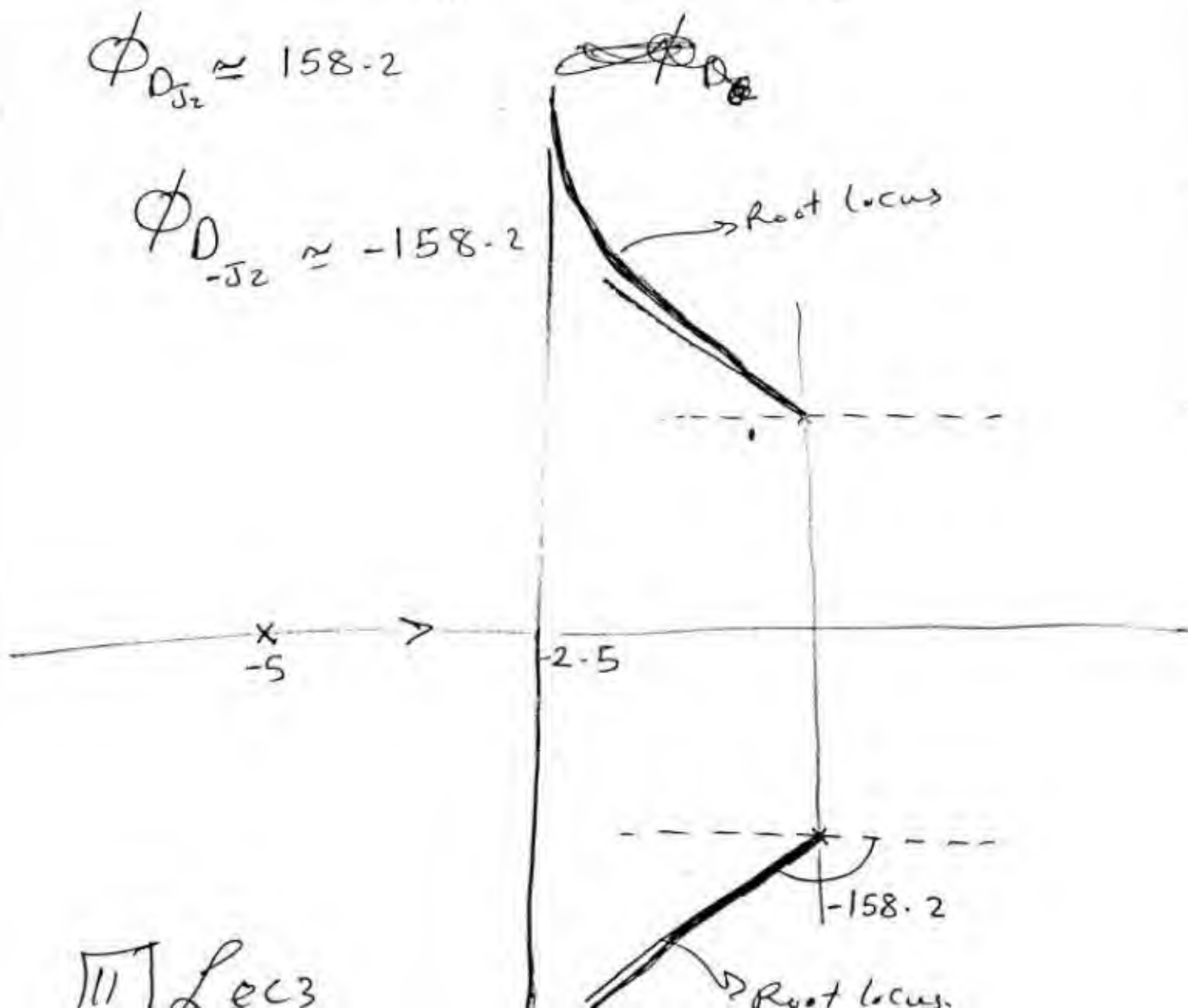


[5] Def. Angle

$$\phi_{D_{J_2}} = 180 - \underbrace{\left( 90 + \tan^{-1}\left(\frac{2}{5}\right) \right)}_{\phi_{p_2}} + \underbrace{90}_{\phi_z}$$

$$\phi_{D_{J_2}} \approx 158.2$$

$$\phi_{D_{-J_2}} \approx -158.2$$



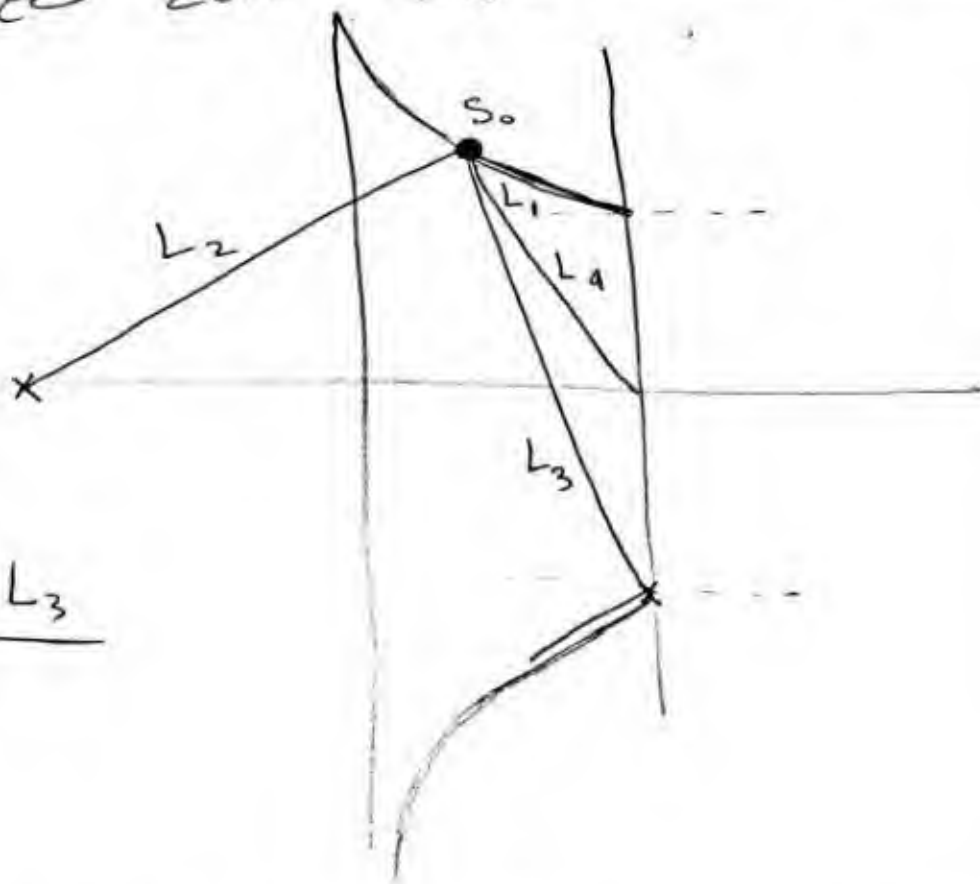
III Lec 3

1] Find K at  $s_0$

(root locus)  $\rightarrow s_0$  نقطة تقع على ال

$$K|_{s_0} = \frac{\prod \text{Poles}}{\prod \text{Zero}}$$

$\leftarrow$  في حالة إذا لم يوجد zero نضع مكانه 1



$$K|_{s_0} = \frac{L_1 \cdot L_2 \cdot L_3}{L_4}$$

2] K at  $\zeta = 0.5$

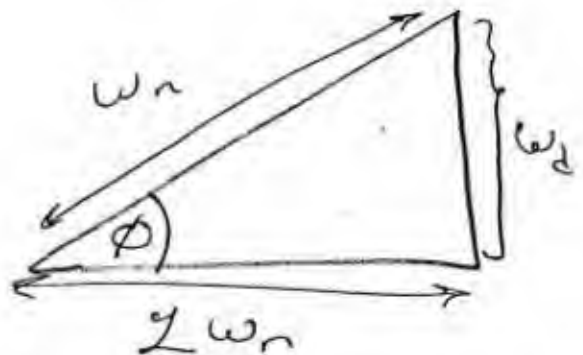
$\zeta \rightarrow$  damping ratio

$$T.F = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$S_{1,2} = -\zeta \omega_n \pm j \omega_d$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

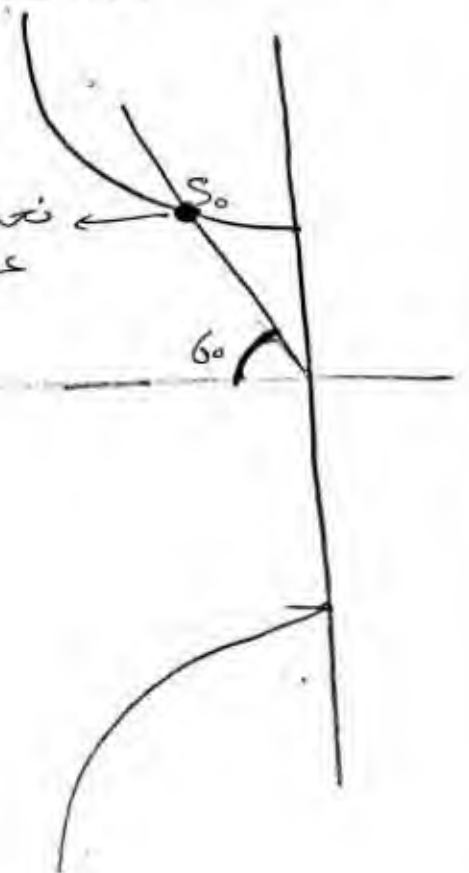
$$\phi = \cos^{-1} \zeta = 60^\circ$$



مع بنقسي زاوية مع المحور الأفقي  
بالمثل

لو عملت الزاوية مع تحت  
هتطلع نفس الشكل.

نحسب ~~الزاوية~~  $K$   
عند النقطة دي.



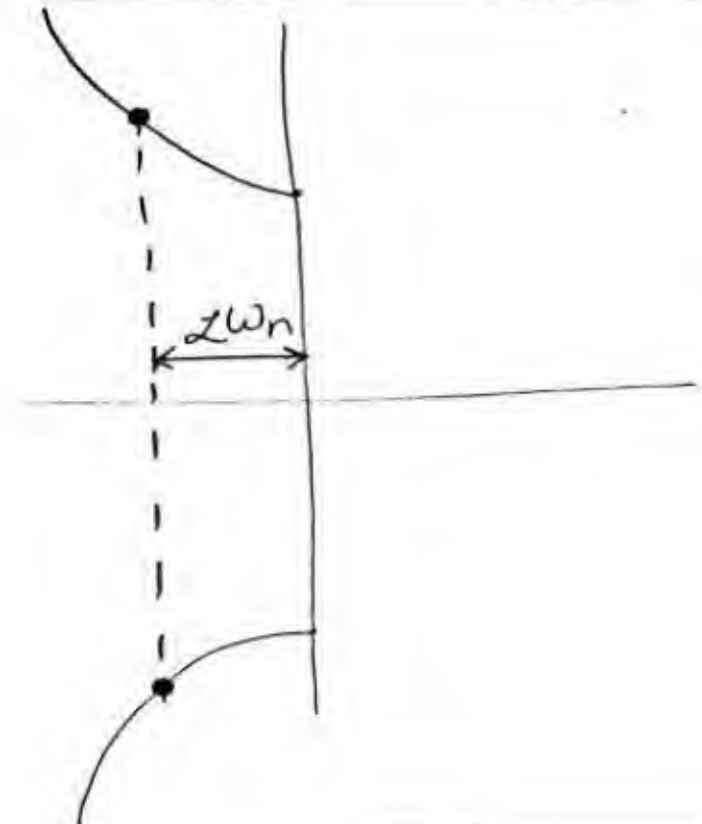
[3] Find  $K$  at  $t_s = \checkmark$  sec.

Given  $2\omega_n$

$$t_s = \frac{4}{2\omega_n} \quad (2\% \text{ error})$$

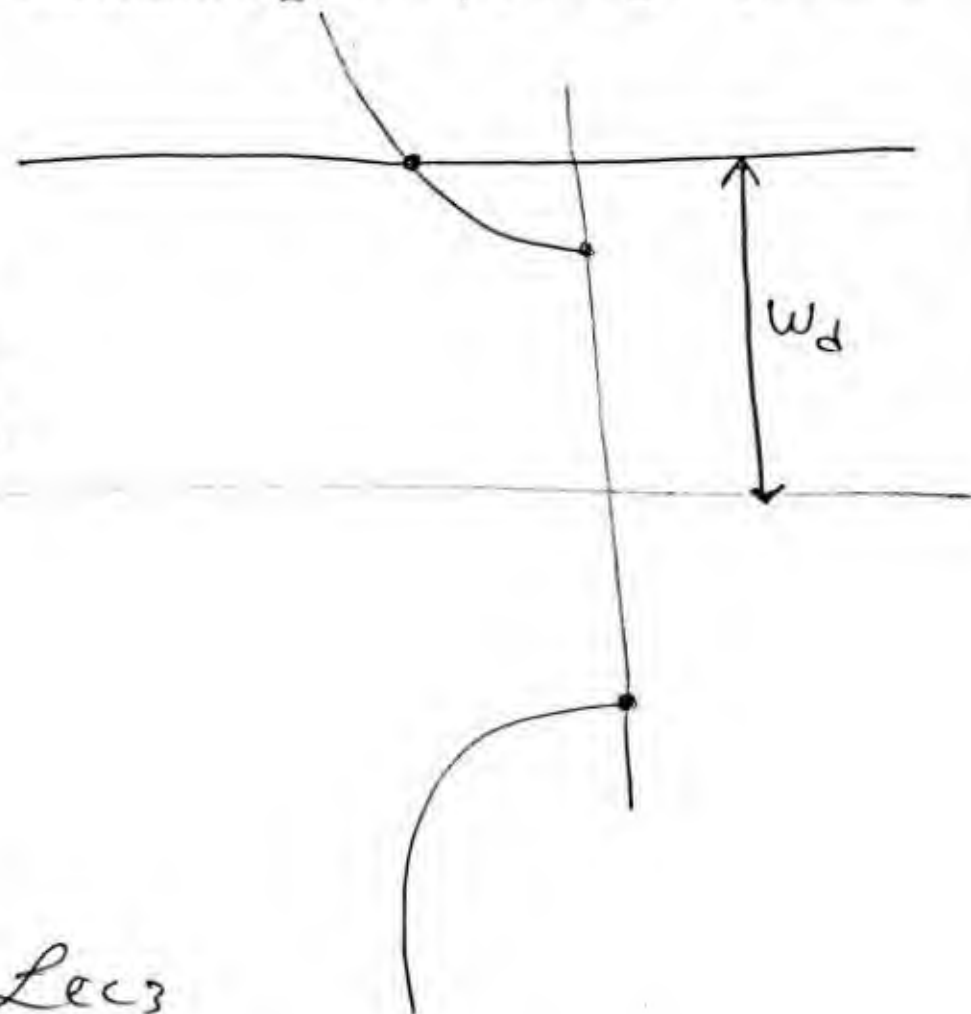
~~هتطلع نفس الشكل~~

خط فاصل منقسمين  
 خطوط منقسمين  $Zero, Pole$   
 واحسب  $K$  كفاي رقم (1)



**[4]**  $K$  at  $\omega_d = \dots$   
 $\rightarrow$  damping frequency (rad/sec)

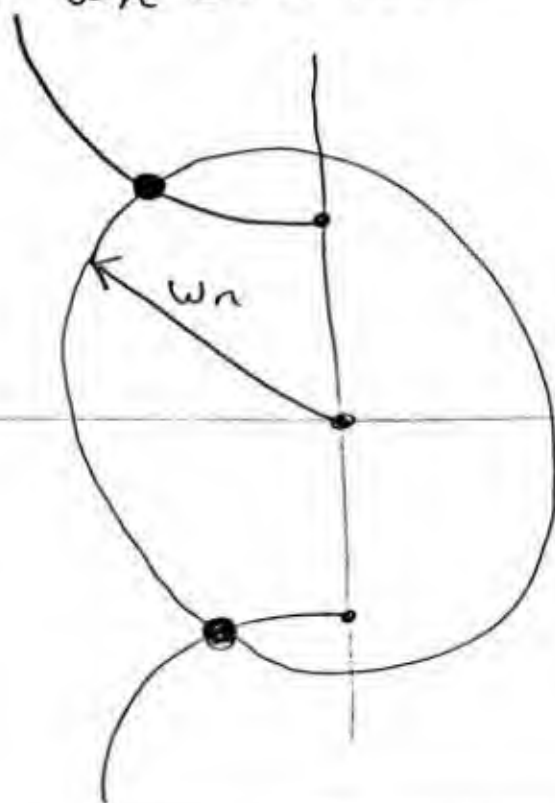
البعد عن الأفقي مسافة  
 $(\omega_d)$  لأعلى أو لأسفل  
 ولرسم خط أفقي صيقل  
 مع ال (root locus)



**[14]** Lec 3

5 Find  $K$  at  $\omega_n = 8 \text{ rad/sec}$

منه عند نقطة الأصل  
تقسم دائرة ديفت قطرها  
يساري  $(\omega_n)$  مستطاع  
مع ال (root locus)

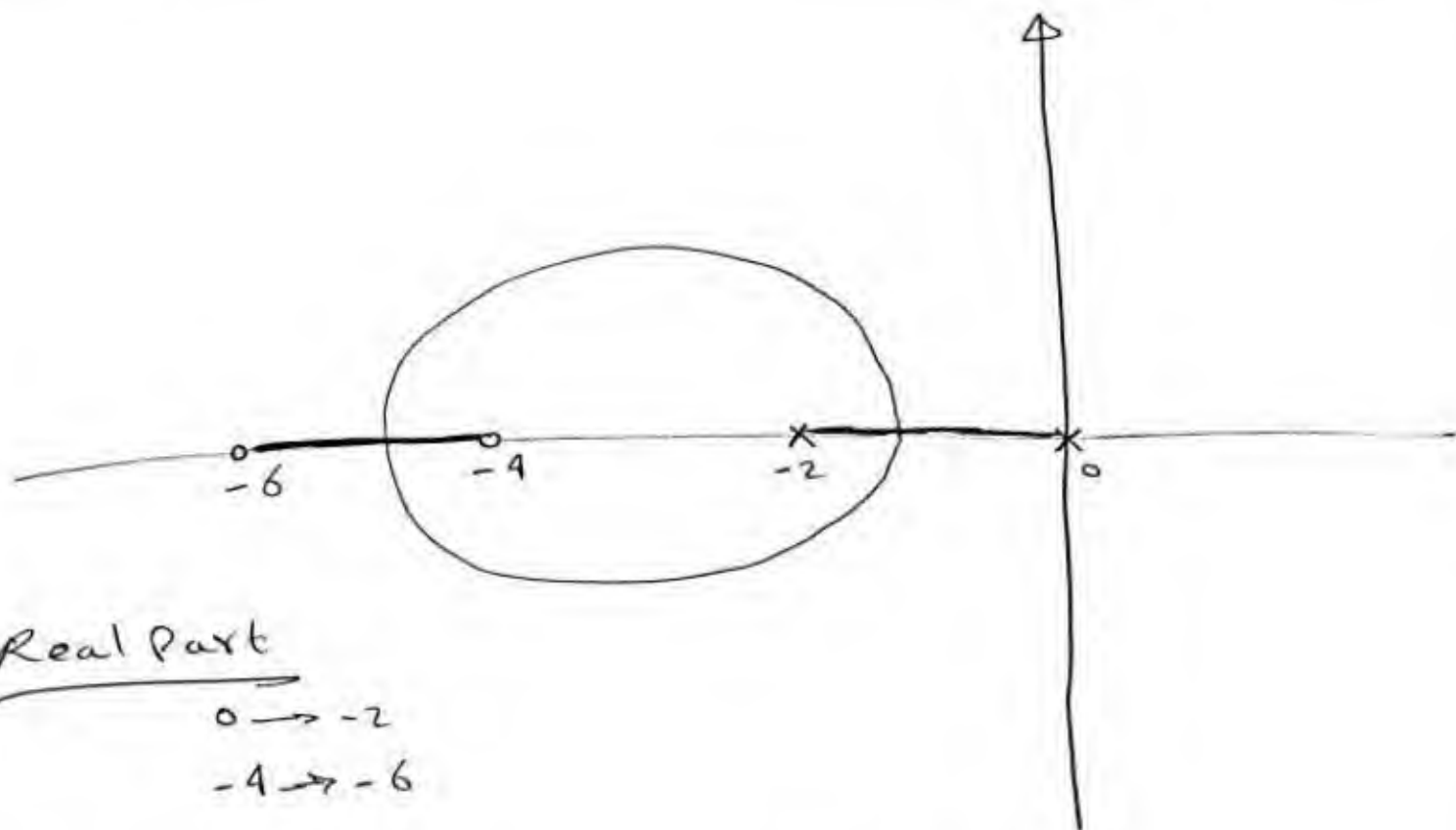


Report

$$GH(s) = \frac{K(s+4)(s+6)}{s(s+2)}$$

- Draw root locus
- Find the range of  $K$  for stability.
- $K$  at  $t_s = 2 \text{ sec} = \frac{4}{2\omega_n} \Rightarrow 2\omega_n = 2$
- $K$  at  $Z = \frac{\sqrt{3}}{2} \Rightarrow \phi = \cos^{-1} Z = 30^\circ$

على ورقه رسم بياني .



Real Part

0  $\rightarrow$  -2

-4  $\rightarrow$  -6

Breaking point

نقطه انفصال دائرة

← ارسم الدائرة على محور رسم بياني .

← لو كان شكل المثال

$$GH(s) = \frac{10K(s+4)(s+6)}{s(s+2)}$$

Put  $10K = K'$

16 Lec 3